

DEEP INELASTIC ELECTROPION PRODUCTION*

A. Calogeracos

School of Engineering

University of Thessaly

Volos 383 34, Greece

Norman Dombey[†]

Physics Division

University of Sussex

Brighton, Sussex

United Kingdom BN1 9QH

Geoffrey B. West[‡]

High Energy Physics, T-8, MS B285

Los Alamos National Laboratory

Los Alamos, NM 87545

U. S. A.

*Research supported by NATO Science & Environmental Affairs Division and
The DOE High Energy Physics Division

[†] (normand@syma.sussex.ac.uk)

[‡] (gbw@pion.lanl.gov)

ABSTRACT

This paper is devoted to a study of possible scaling laws, and their logarithmic corrections, occurring in deep inelastic electropion production. Both the exclusive and semi-exclusive processes are considered. Scaling laws, originally motivated from PCAC and current algebra considerations are examined, first in the framework of the parton model and QCD perturbation theory and then from the more formal perspective of the operator product expansion and asymptotic freedom, (as expressed through the renormalization group). We emphasize that these processes allow scaling to be probed for the full amplitude rather than just its absorptive part (as is the case in the conventional structure functions). Because of this it is not possible to give a formal derivation of scaling for deep inelastic electropion production processes even if one believes that they are unambiguously sensitive to the light cone behavior of the operator product. The origin of this is shown to be related to its behavior near $x \approx 0$. Investigations, both theoretical and experimental, of these processes is therefore strongly encouraged.

1. INTRODUCTION

1.1. *Motivation*

The well-known logarithmic scaling violations in the structure functions of nucleons predicted by asymptotic freedom played a crucial role in establishing QCD as the accepted theory of strong interactions. These predictions, based as they are on the renormalization group and the operator product expansion of two electromagnetic currents near the light cone, are on a strong theoretical footing. This is in contrast to the situation in many other hadronic processes. For example, a theoretical analysis of the Drell-Yan process, or of jet production, requires some input of unknown soft infrared contributions [1]. In spite of this ignorance, theorists concerned with these processes have gone ahead with recipes for their calculation, which often have been very successful. These recipes involve a careful mix of several ingredients, some of which such as asymptotic freedom and perturbation theory are well-understood, while others, such as the ingredient describing non-perturbative hadronization effects are chosen with an eye for the acceptability of the finished product rather than for their theoretical basis. Nevertheless this pragmatic approach is now taken to be sufficiently reliable that it is used to estimate backgrounds in experiments searching for unusual or new phenomena: for example calculations of jet production are used in searches for potential Higgs candidates.

It is therefore important for theorists to appreciate the lack of knowledge which is a necessary input into such calculations involving perturbative QCD so that experimental results can be used to illuminate the approximations and assumptions used in the recipe. It is in this spirit that we consider here another hadronic process which is similar in many respects to the classic deep inelastic structure function process which underpins perturbative QCD. This is the process of deep inelastic exclusive pion production from nucleons which each of us worked on many years ago using the ideas of current algebra and PCAC. Our original analyses were clearly not rigorous but they did lead to predictions of scaling which agree with experiment albeit at relatively low energies. We therefore re-analyse this process from a modern viewpoint in this paper: we derive the scaling laws and calculate the expected logarithmic deviations from scaling.

It seems to us that it is important to stimulate more interest in this and similar processes at this time because they are amenable to experiment in the not-too-distant future at HERA, CEBAF and a possible upgrade of SLAC. Much of what we say is applicable also to other processes which are described theoretically by an amplitude which is proportional

to the product of two currents. This should therefore include the deep inelastic electroproduction of ρ 's, K 's and most interestingly real photons: the latter process corresponding to a direct measurement on the non-forward Compton amplitude, albeit with one virtual and one real photon. We will show that if the predictions of scaling behaviour are indeed verified by experiment, important implications follow about the analytic structure of the amplitudes for these processes.

In section 1.2 we define the process we are interested in and the corresponding kinematic region. In section 2.1 we outline the derivation of the scaling law as was done originally by two of us over "twenty" years ago [2] [3] using Current Algebra. The resulting predictions were, in fact, verified by some rather coarse data taken about that time [4] [5]. In section 2.2 we present a rederivation of the scaling law using the language of the parton model as done in the thesis of one of us in the early 1980's [6]. In section 2.3 we calculate logarithmic corrections to the amplitude using the diagrammatic approach pioneered in the seminal paper by Altarelli and Parisi [7]. It should be noted that the end result of this approach is the integral for the amplitude M , which is analogous to the Altarelli-Parisi evolution equation. This equation can then be used to predict the moments of the amplitude, which in principle can be measured. The validity of the procedure is subject to some reservations which we will discuss fully. In section 3 we approach the problem from the point of view of Operator Product Expansion. We explain why the prediction for the moment of the amplitude is sensitive to the analytic properties of the amplitude near $x=0$. This implies that an experimental study of deep inelastic pion production from which these moments can be determined may well yield information on the low x dependence of the amplitude. In section 4 we discuss the connection of our results to experimental data and suggest future experiments.

1.2. Kinematics & Definitions

The amplitude that we are going to study is defined as follows:

$$M_\mu = \langle p' \pi | J_\mu | p \rangle \quad (1.1)$$

$$= (m_\pi^2 - q'^2) \int d^4x e^{iq \cdot x} \langle p' | \theta(x_0) [J_\mu(x), \phi_\pi(0)] | p \rangle \quad (1.2)$$

$$= \frac{(m_\pi^2 - q'^2)}{f_\pi m_\pi^2} \int d^4x e^{iq \cdot x} \langle p' | \theta(x_0) [J_\mu(x), \partial^\nu A_\nu(0)] | p \rangle \quad (1.3)$$

Here $J_\mu(x)$ is the electromagnetic current, $A_\mu(x)$ the axial current, m_π the pion mass, f_π its decay coupling constant and $\phi_\pi(x)$ its field. In going from (1.2) to (1.3), the standard PCAC identification has been used:

$$\partial_\mu A^\mu(x) = f_\pi m_\pi^2 \phi_\pi(x). \quad (1.4)$$

The kinematics are illustrated in fig. 1: p is the 4-momentum of the struck target, p' its final momentum and q that of the virtual photon delivered by the scattered electron; q' will be used for the pion 4-momentum.

The relationship to, and generalization from the amplitude probed by measuring the conventional structure functions is clear. In that case one is probing only the imaginary part of the forward Compton amplitude whereas in electropion production one measures a full amplitude which, in general, is non-forward. Formally, the difference can be expressed as probing the difference between a time-ordered, or retarded product, as in (1.3), versus a commutator as in the structure function case:

$$W_{\mu\nu} = \int d^4x e^{iq \cdot x} \langle p | [J_\mu(x), J_\nu(0)] | p \rangle. \quad (1.5)$$

The full forward Compton amplitude is given by

$$\mathcal{J}_{\mu\nu} \equiv \int d^4x e^{iq \cdot x} \langle p | T[J_\mu(x), J_\nu(0)] | p \rangle \quad (1.6)$$

so that $W_{\mu\nu} = \text{Im } \mathcal{J}_{\mu\nu}$. These are represented by the diagrams in fig. 2.

A further crucial difference between the two cases is, of course, that in (1.3), the kinematics of real pion production dictates that, even in the deep inelastic limit when q^2 is large, q'^2 must remain fixed at m_π^2 ; on the other hand, in (1.5), the magnitude of the virtual mass of both currents is always large in the deep inelastic limit. This latter condition ensures that the light-cone is unambiguously being probed and so justifies the use of the light cone operator product expansion. In spite of the fact that this is not clearly the case in pion production we shall argue below that a short distance operator product expansion may dominate the process when q^2 is large.

There is a subtlety in this procedure which is also present in the standard forward Compton amplitude case. The point is that this formalism leads to an expansion in powers of $1/x$, where $x \equiv -q^2/2p \cdot q$, and, in the physical region accessible to real experiments, $|x| < 1$. Such an expansion therefore clearly does not converge. In the structure function case, this potential problem is finessed because, there, one is interested in only the imaginary

part, as in (1.5), so an analytic continuation from the unphysical large $|x|$ (where the expansion presumably makes sense) to the physical region can be effected [8]. Indeed this is why the results are expressed in the form of moments of the structure functions rather than the structure functions themselves. We shall discuss this in more detail below. However, it is clear from this, that in the pion production case M_μ is sensitive to a potentially interesting part of the formalism not readily accessible to the structure functions. Indeed it may well be that because of this “problem” pion electroproduction can cast interesting light on the general small x behavior as well as the general assumptions that underly the usual derivation. Before reviewing the old scaling arguments, however, let us recall the relationship between the measured cross-section and the matrix element M_μ : this is best done in terms of the tensor

$$T_{\mu\nu} \equiv M_\mu M_\nu^*. \quad (1.7)$$

The result is given by: [9]

$$\frac{d^3\sigma}{dE'd\Omega'd\Omega} = \frac{\alpha}{2\pi^2q^2} \frac{E'}{E} \frac{(v^2 - q^2)^{1/2}}{1 - \epsilon} \frac{d\sigma}{d\Omega}. \quad (1.8)$$

where $E(E')$ is the initial (final) energy of the electron in the Laboratory (LAB) system and ν its energy loss: note that $\nu = E - E' = p \cdot q / M$ where M is the target nucleon mass. The polarization of the virtual photon is given by:

$$\epsilon = [1 - \frac{2(\nu^2 - q^2)}{q^2} \tan^2 \frac{1}{2}\theta_e]^{-1} \quad (1.9)$$

where θ_e is the electron scattering angle in the LAB. The quantity $d\sigma/d\Omega$ represents an equivalent virtual photoproduction cross-section in the outgoing hadron center-of-mass (CM) system:

$$\begin{aligned} \frac{d\sigma}{d\Omega} = & \frac{M^2|q'|_{\text{CM}}}{16\pi^2W^2|q|_{\text{CM}}} \left[\frac{1}{2}(T_{xx} + T_{yy}) + \frac{1}{2}\epsilon(T_{xx} - T_{yy}) \right. \\ & \left. - (q^2/\nu^2)\epsilon T_{zz} + \{-(2q^2/\nu^2)\epsilon(1 + \epsilon)\}^{\frac{1}{2}} T_{xz} \right] \end{aligned} \quad (1.10)$$

Here W is the total CM energy so $W^2 \equiv s = (p + q)^2 = (p' + q')^2$. The z -axis is defined to be coincident with the direction of q whilst the electrons define the xy plane. Thus all of the ϕ (azimuthal) dependence is contained in $(T_{xx} - T_{yy}) \sim \cos 2\phi$ and $T_{xz} \sim \cos \phi$. In what follows we shall limit ourselves to the case where the particle spins are unobserved.

Finally, it is worth noting that in the deep inelastic limit ($q^2 \rightarrow -\infty$), the square of the momentum transfer

$$t \equiv \Delta^2 \equiv (q' - q)^2 = (p' - p)^2 \quad (1.11)$$

is constrained, in the physical region, to lie between -2ν and

$$t_{\min} \approx -\frac{x^2 M^2}{(1-x)}. \quad (1.12)$$

Typically the limit we will be considering keeps t and x fixed (and finite) with $q^2 \rightarrow -\infty$. Thus x must not be too close to unity. Furthermore, the region of interest is predominantly forward scattering in the πN CM system. In what follows it is convenient to write

$$q_\mu = E n_\mu + \Delta_\mu \quad (1.13)$$

where n^2 is a null-vector, i.e. $n^2 = 0$. Thus

$$q^2 = 2E(n \cdot \Delta) + t \quad (1.14)$$

and

$$x = \frac{-[2E(n \cdot \Delta) + t]}{(2[E(n \cdot p) + \Delta \cdot p])}. \quad (1.15)$$

The scaling limit can then be realized by taking $E \rightarrow \infty$ with both $x \approx -(n \cdot p)/(n \cdot \Delta)$ and t fixed.

2. SCALING LAW

2.1. Current Algebra

Scaling laws for $T_{\mu\nu}$ can be derived using a current algebra approach augmented by some heuristic assumption about the light cone behavior of the commutator. This can be checked in perturbation theory and justified by the operator product expansion as sketched below. We begin by setting $q'^2 = 0$ in which case

$$f_\pi M_\mu = C_{\mu\nu} q'^\nu + E_\mu \quad (2.1)$$

where

$$C_{\mu\nu} \equiv i \int d^4x \exp(iq \cdot x) \langle p' | \theta(x_0) [J_\mu(x), A_\nu(0)] | p \rangle \quad (2.2)$$

and

$$E_\mu \equiv \int d^4x \exp(iq \cdot x) \langle p' | \delta(x_0) [J_\mu(x), A_0(0)] | p \rangle. \quad (2.3)$$

Using the usual $SU(2) \times SU(2)$ current algebra

$$\delta(x_0) [A_0^i(x), J_\mu(0)] = i\epsilon^{i3k} A_\mu^k(0) \delta^4(x) \quad (2.4)$$

we immediately get that E_μ is independent of E ; (it depends only on Δ and, from its usual parametrization, we get the well-known axial vector and induced pseudoscalar form factors of the nucleon).

The scaling result we want to show is that $C_{\mu\nu}q'^\nu$ is also independent of E . We shall first sketch the derivation of this result based on the spacetime behavior of the current commutators. In what follows it is convenient to introduce standard light-cone coordinates for a four-vector a^μ as follows:

$$a_\pm = \frac{\sqrt{2}}{2}(a_0 \pm a_z); \quad \mathbf{a}_\perp = (a_x, a_y). \quad (2.5)$$

Then the scaling limit is equivalent to $q_- \approx \sqrt{2}\nu \rightarrow \infty$ with $q_+ \approx \sqrt{2}x$ fixed.

Causality allows us, at least naively, to replace $\theta(x_0)$ in (2.2) by $\theta(x_+)$ in which case the asymptotic behavior of $C_{\mu\nu}$ is given by

$$C_{\mu\nu} \approx -\frac{1}{q_-} \int d^4x \exp(iq \cdot x) \langle p' | \delta(x_+) [J_\mu(x), A_\nu^i(0)] | \rangle \quad (2.6)$$

Consequently,

$$q'^\nu C_{\mu\nu} \sim q_- C_{\mu+} \sim - \int d^4x \exp(iq \cdot x) \langle p' | \delta(x_+) [J_\mu(x), A_+^i(0)] | p \rangle. \quad (2.7)$$

The commutator in (2.7) can be expressed in the form

$$[J_\mu(x), A_+^i(0)] \delta(x_+) = \tilde{A}_\mu^i(x_-) \delta^2(x_\perp) \quad (2.8)$$

where $\tilde{A}_\mu^i(x_-)$ is, in general, model-dependent.

In QCD, canonical commutation relations lead to

$$\tilde{A}_\mu^i(x_-) = A_\mu^i(0) \delta(x_-) + B_\mu^i(x_-) \quad (2.9)$$

where $B_\mu^i(x_-)$ is an unknown (typically bilinear non-local) operator which is non-singular at $x_- \approx 0$. We conclude then that, in the scaling limit,

$$q'^\nu C_{\mu\nu} \sim \int dx_- \exp(iq_+ x_-) \langle p' | B_\mu^i(x_-) | p \rangle. \quad (2.10)$$

This is the desired result since it shows that M_μ is independent of E and is only a function of x (through q_+) and Δ .

This result straightforwardly translates into the following scaling constraints on the components of $T_{\mu\nu}$ occurring in the measured cross-section, (1.8):

$$\begin{aligned} \frac{1}{2}(T_{xx} + T_{yy}) &\rightarrow F_1(x, t)(k_x^2 + k_y^2) \\ \frac{1}{2}(T_{xx} - T_{yy}) &\rightarrow \frac{1}{2}F_2(x, t)(k_x^2 - k_y^2) \\ T_{zz} &\rightarrow F_1(x, t) - F_2(x, t)\Delta_z^2 \\ \text{and } T_{xz} &\rightarrow -F_2(x, t)k_x\Delta_z \end{aligned} \quad (2.11)$$

where Δ_z (as expressed in the LAB) $\approx -(\frac{1}{2}t + M\omega)$ and the F_i are Lorentz scalars.

2.2. Parton Model

We now turn to the treatment of this problem in terms of the quark-parton model. We assume that the two currents J_μ and A_ν interact successively with the same quark while the rest act as spectators. Without QCD corrections the amplitude $C_{\mu\nu}$ is given by the sum of the two diagrams in fig. 3. This is analogous to the usual parton model treatment of the forward Compton scattering.

To calculate these diagrams we assume that the struck quark carries a fraction η of the momentum p of the hadron, in the infinite momentum frame. Immediately after absorbing the photon the virtual quark has momentum $(\eta p + q)$, and hence it is highly off-shell. This is the basic reason for considering that the parton model is applicable here. The final quark has momentum $(\eta p + \Delta)$. Consider now the diagram in fig. 3(a). Its contribution is (τ_i are isospin matrices and \tilde{Q} is the generator corresponding to the electric charge):

$$\begin{aligned} M_{(a)} &= -\frac{\tau_i}{2} \tilde{Q} \frac{\tilde{\psi}_{p'}(\eta p + \Delta) \gamma_5 \gamma_\nu (\eta \not{p} + E \not{h} + \Delta) \gamma_\nu \psi_p(\eta p)}{2E[\eta(n.p) + (n.\Delta)]} q'_\nu \\ &= -\frac{\tau_i}{2} \tilde{Q} \frac{\tilde{\psi}_{p'}(\eta p + \Delta) \gamma_5 \not{h} (\eta \not{p} + \Delta) \gamma_\mu \psi_p(\eta p)}{2[\eta(n.p) + (n.\Delta)]} \end{aligned} \quad (2.12)$$

To this has to be added the contribution from the crossed graph shown in fig. 3(b). A sum over the various types of quarks has been suppressed. $\psi_p(k)$ represents the amplitude, or wave function, for finding a quark of a particular type carrying momentum k inside a nucleon moving with momentum p . The complete matrix element requires an integration over η , consistent with the requirement that $(\eta p + q)^2 > 0$ i.e. $\eta > x$. Schematically, the parton contribution is thus given by

$$\mathcal{M}_\mu^0 = \int_x^1 d\eta \bar{\psi}_{p'}(\eta p + \Delta) M_\mu(\eta, p, \Delta) \psi_p(\eta p). \quad (2.13)$$

The matrix \mathcal{M}_μ can be read off from (2.13) with an additional contribution coming from the crossed graph. This shows explicitly that, when E is large, M depends only on t and x .

Note, incidentally, that for the parton model description of the conventional structure function, $\Delta = 0$ and only the imaginary part of \mathcal{M} , i.e. its delta-function contribution, is required. In that case the integral reduces to $xf(x)$, where $f(\eta) \equiv \bar{\psi}_p(\eta p) \psi_p(\eta p)$, the probability for finding a quark with fraction η of the total momentum. It is worth remarking that in the analogous kinematic configuration here, where $\Delta \approx 0$, the full Born amplitude reduces to

$$M_\mu \approx [Q, \frac{\tau^i}{2}](n.p)p_\mu. \quad (2.14)$$

2.3. Leading Logarithmic Corrections

In this section we sketch a computation of the leading logarithmic corrections to the parton model result. Rather than give a complete detailed description we present here only the salient features for the simpler and more limited kinematic situation where $\Delta \approx 0$. The point is that a subgraph which gives a logarithmic contribution $\sim \ln \Delta^2$ for $\Delta \neq 0$ obviously is singular in the $\Delta \rightarrow 0$ limit: hence the same diagrams (i.e. ladders and self-energy insertions to external legs) that give the leading logarithmic corrections in the usual deep inelastic scattering case ($\Delta = 0$) also give the leading logarithmic corrections in the $\Delta \neq 0$ case.

For ease of presentation, we shall from here on use p to denote the momentum of the initial quark rather than of the initial hadron. The initial quark is taken to be off-shell by an amount comparable to the inverse of the confinement radius of the nucleon. Since the momentum transfer Δ is typically of the same order of magnitude, we make no distinction between $\ln(-q^2/p^2)$ and $\ln(-q^2/\Delta^2)$. When, for example, we end up with a logarithmic integration of the form $\int dk^2/(k^2 + \Delta^2)$ we shall be free to take the $\Delta \rightarrow 0$ limit and write it in the form $\int_{p^2} dk^2/k^2$. Although the final scattered quark is slightly off-shell, the $\Delta \rightarrow 0$ limit enables us to take the γ matrices between on-shell spinors. It is important, of course, to take the $\Delta \rightarrow 0$ limit after having secured that the final integration is logarithmic.

It is convenient to work in the light cone gauge where the propagator for a gluon of momentum k is

$$G_{\mu\nu}(k) = \frac{D_{\mu\nu}(k)}{k^2 + i\epsilon} \quad (2.15)$$

with

$$D_{\mu\nu}(k) = g_{\mu\nu} - \frac{k_\mu c_\nu + k_\nu c_\mu}{k \cdot c} . \quad (2.16)$$

In such a gauge the only diagrams (apart from self energy parts) giving leading logarithmic corrections are the ladder diagrams of fig. 4(a). It may be mentioned, in particular, that diagrams of the type of fig. 4(b) do not give leading logarithmic contributions. Incidentally, had we followed the operator product expansion approach these diagrams would correspond to contributions from gauge non-invariant operators.

Turning now to the calculation of the ladder diagrams we show that a familiar picture emerges: there is strong ordering in the momentum flowing through the ladder and an evolution equation can be derived. We first examine the contribution from the lowest order diagram (fig. 5). Using a Sudakov parametrization for the quark momentum k

$$k = \alpha c + \beta p + k_\perp \quad (2.17)$$

$$d^4k = \frac{s}{2} d\alpha d\beta d^2k_\perp^2 \quad (2.18)$$

(where $s = -q^2/x$). We obtain:

$$M_1 = -\frac{\tau i}{2} \tilde{Q} \frac{\alpha_s C_F}{4\pi^3} \frac{s}{4} \int dk_\perp^2 d\alpha d\beta \frac{\gamma_\sigma(\not{k} + \not{\Delta}) \not{\Delta}(\not{k} + \not{\Delta}) \gamma_\mu \not{k} \gamma_\rho \gamma_5}{[(k + E_n + \Delta)^2 + i\epsilon](k^2 + i\epsilon)[(k + \Delta)^2 + i\epsilon][(p - k)^2 + i\epsilon]} D_{\rho\sigma} \quad (2.19)$$

Initial and final spinors have been suppressed. Note that the amplitude is color singlet in the t-channel and that, in terms of Sudakov variables,

$$k^2 = \alpha\beta s - k_\perp^2, \quad (p - k)^2 = -\alpha(1 - \beta)s - k_\perp^2, \quad (k + \Delta)^2 = \alpha\beta s - k_\perp^2 - \alpha s x. \quad (2.20)$$

We first perform the α integration. In the region $x < \beta < 1$ there is one pole at $\alpha = -k_\perp^2/(1 - \beta)s$ due to $(p - k)^2$ lying below the real axis whereas in the region $0 < \beta < x$ there is one pole at $\alpha = k_\perp^2/\beta s$ due to k^2 lying above it. In both cases we close the α contour so as to pick the contributions from those poles. In the regions $\beta < 0$ and $\beta > 1$ all the poles with respect to α lie on one side of the real axis and can be avoided (we do not take $(k + E_n + \Delta)^2$ into account since it will be combined with the next element of the ladder). Observe that the leading logarithms come from the wide range of integration $\mu^2 \leq k_\perp^2 \ll s$ where μ is an arbitrary renormalization scale. The parameter β is finite (typically of order x) whereas $\alpha \sim -k_\perp^2/s$ is small. Hence the logarithms come, as expected, from the collinear configuration.

After the α integration the denominator behaves like $(k_\perp^2)^2$, so we have to extract one k_\perp^2 from the numerator if the final integration is to be logarithmic. Having done this we can pass to the collinear configuration $k = \beta p$, $\alpha \approx 0$. As already remarked we shall also set $\Delta \approx 0$ and take the γ matrices between on-shell spinors $u(p)$. One might worry whether in the limit $\Delta \rightarrow 0$ we lose logarithms multiplied by $q \cdot \Delta/q^2$. There is no such danger since we parametrize everything from the start in terms of vectors n, p and Δ . Then $q \cdot \Delta/q^2 \sim E(n \cdot \Delta)/2E(n \cdot \Delta) \sim \frac{1}{2}$. There will remain a factor $\gamma_5 \not{\Delta} \not{k} \gamma_\mu$ from the numerator which will combine with $(k + E_n + \Delta)^2$ from the denominator to form the Born term $M_B(\beta p, k_\perp^2)$. Note finally that the logarithmic contribution comes from the region $\beta > x$, so that the propagating quark line remains highly off-shell.

The numerator in the integrand has the form (apart from the γ_5):

$$\gamma_\sigma(\not{k} + \not{\Delta}) \not{\Delta}(\not{k} + \not{\Delta}) \gamma_\mu \not{k} \gamma_\rho \left\{ g_{\rho\gamma} - \frac{c_\rho(p - k)_\sigma + c_\sigma(p - k)_\rho}{c \cdot (p - k)} \right\}. \quad (2.21)$$

After some algebra this can be reduced (in the $\Delta \approx 0$ limit) to a familiar form for the leading lowest order correction:

$$M_1(x, q^2) = 2C_F \int_{\mu^2}^{-q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \frac{\alpha_s}{4\pi} \int_x^1 d\beta \left\{ 1 - \beta + \frac{2\beta}{1-\beta} \right\} M_B(x/\beta, \mu^2) \quad (2.22)$$

Iterating this an arbitrary number of times leads to an evolution equation:

$$M(x, q^2) = M(x, \mu^2) + 2C_F \int_{\mu^2}^{-q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \frac{\alpha_s}{4\pi} \int_x^1 d\beta \frac{1+\beta^2}{1-\beta} M(x/\beta, k_{\perp}^2). \quad (2.23)$$

Up to now we have considered skeleton graphs only. When we dress the ladder with vertex and self-energy corrections further leading logarithmic contributions coming from the ultraviolet region are induced. These can be taken into account simply by replacing the “bare” coupling constant α_s in (2.23) by the running coupling constant $\alpha_s(k_{\perp}^2) \approx 4\pi/(\beta \log k_{\perp}^2)$ where $\beta = (11 - 2n_f/3)$ (n_f being the number of flavors). In addition, the second term in (2.23) must be multiplied by the quark wavefunction renormalization constant Z_F in order to cancel the infrared divergences from the soft gluon region and get a gauge invariant result. Z_F in the light-cone gauge has been calculated in a number of places. The final result is

$$M(x, q^2) = M(x, \mu^2) + 2C_F \int_{\mu^2}^{-q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \frac{\alpha_s(k_{\perp}^2)}{4\pi} \int_x^1 d\beta P(\beta) M(x/\beta, k_{\perp}^2) \quad (2.24)$$

where,

$$P(\beta) = \frac{1+\beta^2}{1-\beta} - \delta(1-\beta) \int_0^1 dx \frac{1+x^2}{1-x}. \quad (2.25)$$

Equation (2.24) is somewhat difficult to handle from the phenomenological point of view. If for the moment we disregard the subtleties regarding the low x dependence of M which will be discussed in the following section, then we can disentangle M in (2.24) by taking moments. Defining

$$M_n(q^2) = \int_0^1 dx x^n M(x, q^2) \quad (2.26)$$

we get

$$M_n(q^2) \sim \left(\ln \frac{q^2}{\mu^2} \right)^{d_n} \quad (2.27)$$

where

$$d_n = \frac{C_F}{\beta} \left(1 + 4 \sum_{j=2}^{n+2} \frac{1}{j} \frac{2}{(n+2)(n+3)} \right) \quad (2.28)$$

3. OPERATOR PRODUCT EXPANSION

In this section we discuss an operator product expansion (OPE) analysis of this amplitude. As already intimated, there are some subtleties that prohibit a straightforward prediction for its asymptotic behavior. Before discussing this, however, it is worth reviewing briefly the standard treatment of the conventional structure functions from this viewpoint. In that case the use of perturbation theory to determine the large q^2 behavior can be justified from the application of the OPE to the light cone expansion of $\mathcal{J}_{\mu\nu}$, (1.6).

Explicitly, the time-ordered product of the two currents can be Taylor expanded around $x^2 \approx 0$ in terms of a complete set of operators $O_{mn}^{\mu_1 \dots \mu_n}$:

$$T[J(x)J(0)] \approx \sum_{m,n} c_m(x^2) x_{\mu_1} \dots x_{\mu_n} O_{mn}^{\mu_1 \dots \mu_n}(0). \quad (3.1)$$

[For ease of presentation, the currents are here taken to be scalar]. Equivalently, its Fourier transform is given by

$$\int d^4x e^{iq \cdot x} T[J(x)J(0)] \approx \sum_{m,n} C_{mn}(q^2) \frac{q_{\mu_1} \dots q_{\mu_n}}{(\frac{1}{2}q^2)^n} O_{mn}^{\mu_1 \dots \mu_n} \quad (3.2)$$

where

$$C_{mn}(q^2) \equiv \left(-\frac{i\partial}{\partial \ln q^2} \right)^n \int d^4x e^{iq \cdot x} c_m(x^2). \quad (3.3)$$

On dimensional grounds the $C_{mn}(q^2)$ behave, up to logarithms, like $(q^2)^{-d_c}$ for large q^2 where $d_c = 2d_J - 4 - (d_0 - n)$ is the dimension of C , d_J that of $J(x)$ and d_0 that of the operator $O_{mn}^{\mu_1 \dots \mu_n}$. This is, of course, the origin of the observation that the asymptotic behavior is controlled by the operator having the lowest twist $\tau_0 \equiv d_0 - n$. Notice that these equations are all properties of the current operators (i.e., in the case of interest here, the pion and the virtual photon) and do not depend on the target state. For forward scattering we require the ground state target matrix elements

$$\langle p | O_{mn}^{\mu_1 \dots \mu_n} | p \rangle = A_{mn} p_{\mu_1} \dots p_{\mu_n} + B_{mn} g_{\mu_1 \mu_2} p_{\mu_3} \dots p_{\mu_n} + \dots \quad (3.4)$$

The A_{mn} , B_{mn} etc. are simply numbers characterizing the target. In the contraction of this with (3.2) it is clear that, in the Bjorken limit, terms involving the A_{mn} dominate: one thereby obtains

$$\begin{aligned} \mathcal{J}(x, q^2) &\equiv \int d^4x e^{iq \cdot x} \langle p | T[J(x)J(0)] | p \rangle \\ &\approx \sum_{m,n} \frac{A_{mn} C_{mn}(q^2)}{x^n} \end{aligned} \quad (3.5)$$

The large q^2 behavior of the $C_{mn}(q^2)$ can be determined from the renormalization group using the asymptotic freedom property of QCD. Typically, for the leading twist operator, the C_{mn} are dimensionless and behave like $(\ln q^2)^{-a_{mn}}$ where a_{mn} is determined by the anomalous dimensions of the O_{mn} . Implications for the structure functions, which are the absorptive part of \mathcal{J} , can be obtained using the standard analytic properties of \mathcal{J} . This leads to the well-known result relating the moments of W to $C_{mn}(q^2)$:

$$\begin{aligned} M_n(q^2) &\equiv \int_0^1 dx x^{n-2} [\nu W(x, q^2)] \\ &\approx \sum_m A_{mn} C_{mn}(q^2) \end{aligned} \tag{3.6}$$

The sum over m is, of course, finite and typically contains only a rather small number of terms. QCD therefore gives a specific prediction for the q^2 -dependence of each moment and it is this that has been successfully checked against experiment [8].

Now, suppose that experiments could be performed that directly measure the large- q^2 behavior of the full amplitude $\mathcal{J}(x, q^2)$. What is the QCD prediction for this? One immediately sees the difficulty: the expansion, (3.5), presumably only makes sense for $|x| > 1$ and this is outside of the physical region. Indeed the analytic continuation to $|x| < 1$ ultimately leads to the moment equations, (3.6). Ideally, one would like to have a complementary expansion valid for $|x| < 1$; this would require knowledge of the analytic structure near $x \simeq 0$ which, unfortunately is not reliably determined by the RG. Naively, one could proceed with \mathcal{J} just as one proceeded with the M_n ; i.e., simply take $q^2 \rightarrow \infty$ in (3.6) and pick out the dominant $C_{mn}(q^2)$ as determined by the smallest anomalous dimension. In the singlet case, for example, the conservation of the stress-energy tensor means that it has no anomalous dimension and so $M_2(q^2)$ asymptotically approaches a constant. This, in turn, means that the leading behavior of the $T_{1,2}(q^2, x)$, the two conventional scalar amplitudes occurring in the decomposition of $\mathcal{J}_{\mu\nu}$, is given by

$$T_1(q^2, x) \approx \frac{T_2(q^2, x)}{2x} \approx \frac{\langle Q^2 \rangle}{\pi q^2 x} \left(\frac{3n_f}{16 + 3n_f} \right) \tag{3.7}$$

Now let us examine the extension of this to the non-forward case. Eqns. (3.1) - (3.3) remain valid since they are properties of the currents and the expansion (3.1) is supposed

to be in terms of a complete set of operators; (3.4) however, clearly needs to be generalized. This can be straightforwardly accomplished by writing:

$$\begin{aligned} \langle p' | O_{mn}^{\mu_1 \cdots \mu_n} | p \rangle = & \sum_{k=0}^n [A_{mnk}(t) p_{\mu_1} \cdots p_{\mu_k} \Delta_{\mu_{k+1}} \cdots \Delta_{\mu_n} \\ & + B_{mnk}(t) g_{\mu_1 \mu_2} p_{\mu_3} \cdots p_{\mu_k} \Delta_{\mu_{k+1}} \cdots \Delta_{\mu_n} + \cdots] \end{aligned} \quad (3.8)$$

Clearly $A_{mnn}(0) = A_{mn}$ and $B_{mnn}(0) = B_{mn}$. When contracting this with (3.2) we shall need the quantity:

$$\frac{2\Delta \cdot q}{-q^2} = 1 + \frac{t}{q^2}. \quad (3.9)$$

It is, therefore, the wider set of coefficients $A_{mnk}(t)$ that dominate the asymptotic behavior: (3.5) is thereby generalized to

$$\mathcal{J}(x, q^2, t) \approx \sum_{n=0}^{\infty} \sum_m \sum_{k=0}^n \frac{A_{mnk}(t) \tilde{C}_{mn}(q^2)}{x^k}. \quad (3.10)$$

The $\tilde{C}_{mn}(q^2)$ are the coefficients appropriate to the axial current case of interest here, as expressed in (1.3) and (2.2), and are the analogs of the $C_{mn}(q^2)$ of (3.5). They, too, generally fall with q^2 like powers of $\ln(q^2)$ determined by the appropriate anomalous dimension. PCAC ensures that, in M_μ , there is an operator with vanishing anomalous dimension so that it becomes a function of q^2 and t only. In any case, the corrections to this will, as usual, be powers of $\ln(-q^2)$. As already explained it is not possible, beyond this, to give the precise prediction for the large q^2 behavior without summing the series.

Finally, it should be noted that the result expressed in (3.10) is clearly not valid unless $t \ll q^2$, which means that x must not be too close to 1. On the other hand, probing scaling and its violation should shed some light on the $x \approx 0$ region: if the predictions of this paper are experimentally verified it means that the relevant amplitudes are smooth in the region of small x where it would appear that the operator product expansion breaks down.

4. COMPARISON WITH EXPERIMENT

In this Section we begin by reviewing the connection of our results to the existing experimental data. At present, the only such data is for the inclusive reaction (and this was taken over 20 years ago at Cornell). It is possible, however, to extend the above arguments to this case provided the mass of the final hadronic “target” state (W') remains

relatively small. The main difference is that the scaling function will now depend on W' in addition to x and t so that, instead of

$$s^2 \frac{d\sigma}{dt} \approx F(x, t) \quad (4.1)$$

which is the result implied by (2.11) for the purely exclusive case, one now expects for the inclusive

$$s^2 \frac{d^2\sigma}{dt dW'^2} \approx F(x, t, W'^2). \quad (4.2)$$

In the Cornell experiment, $d^2\sigma/dtdW'^2$ was measured at two different values of \sqrt{s} (2.66 and 3.14 GeV) but at the *same* value of x . The data was averaged over θ and ϕ . The scaling result, (4.2), implies that the spectra, when plotted as a function of W' , should be identical apart from a normalization factor $(3.14/2.66)^4 \approx 2.41$. The data, as can be readily seen in fig. 6 are in remarkably good agreement with this prediction.

These data are also presented in terms of the transverse momentum P_\perp , the transverse momentum of the pion relative to the direction of the incoming virtual photon and of a variable x' which depends on the longitudinal momentum of the pion:

$$x' = \frac{P_{11}}{(P_{\max}^2 - P_\perp^2)^{\frac{1}{2}}} \quad (4.3)$$

Here P_{11} is the pion momentum along the direction of the virtual photon and P_{\max} is the maximum pion momentum. In fig. 7

$$\frac{E}{\sigma_{\text{tot}}} \frac{d^3\sigma}{dP^3} \quad (4.4)$$

is plotted as a function of P_\perp^2 at the two values of W mentioned previously and at two different values of x' . The straight lines are fits of the form $A \exp(-BP_\perp^2)$. The similarity of the spectra suggests that the P_\perp^2 distribution (for fixed x) does not depend on q^2 , again in striking agreement with the scaling argument.

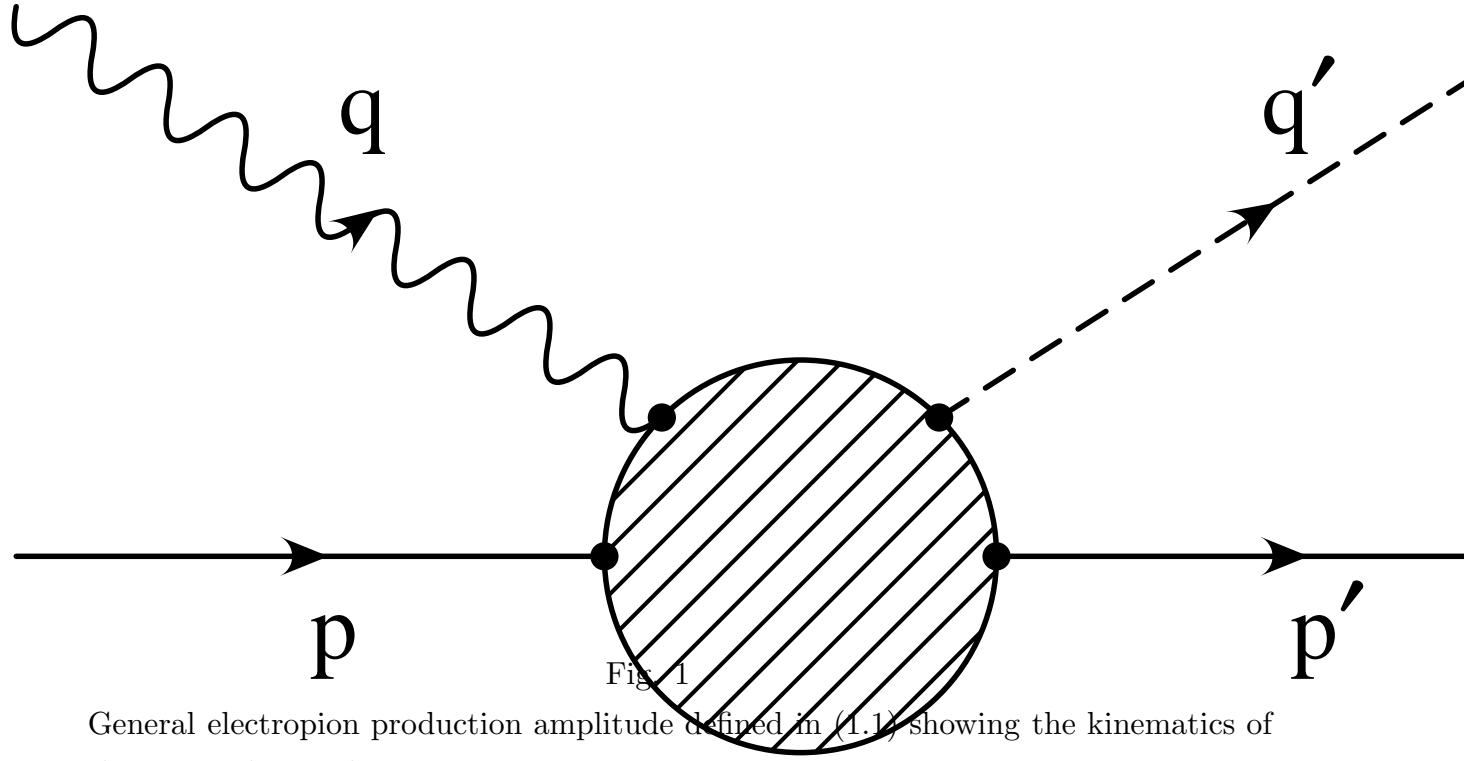
It is clearly important for new experiments to be carried out at HERA on deep inelastic pion electroproduction at high energies check whether scaling continues to hold subject to the logarithmic violations which follow from (2.24) Furthermore the arguments of this paper do not only apply to deep inelastic pion electroproduction. A similar argument could be made for deep inelastic electroproduction of any particle which couples to a nucleon by means of a local current operator. This would include deep inelastic electroproduction of real photons, or of lepton pairs, or of ρ 's, or K 's, or ψ 's, or Υ 's, for example. It would be especially interesting to check whether the amplitude for the production of each of these particles scales in the same way, or whether processes which involve heavy quarks are different.

5. CONCLUSION

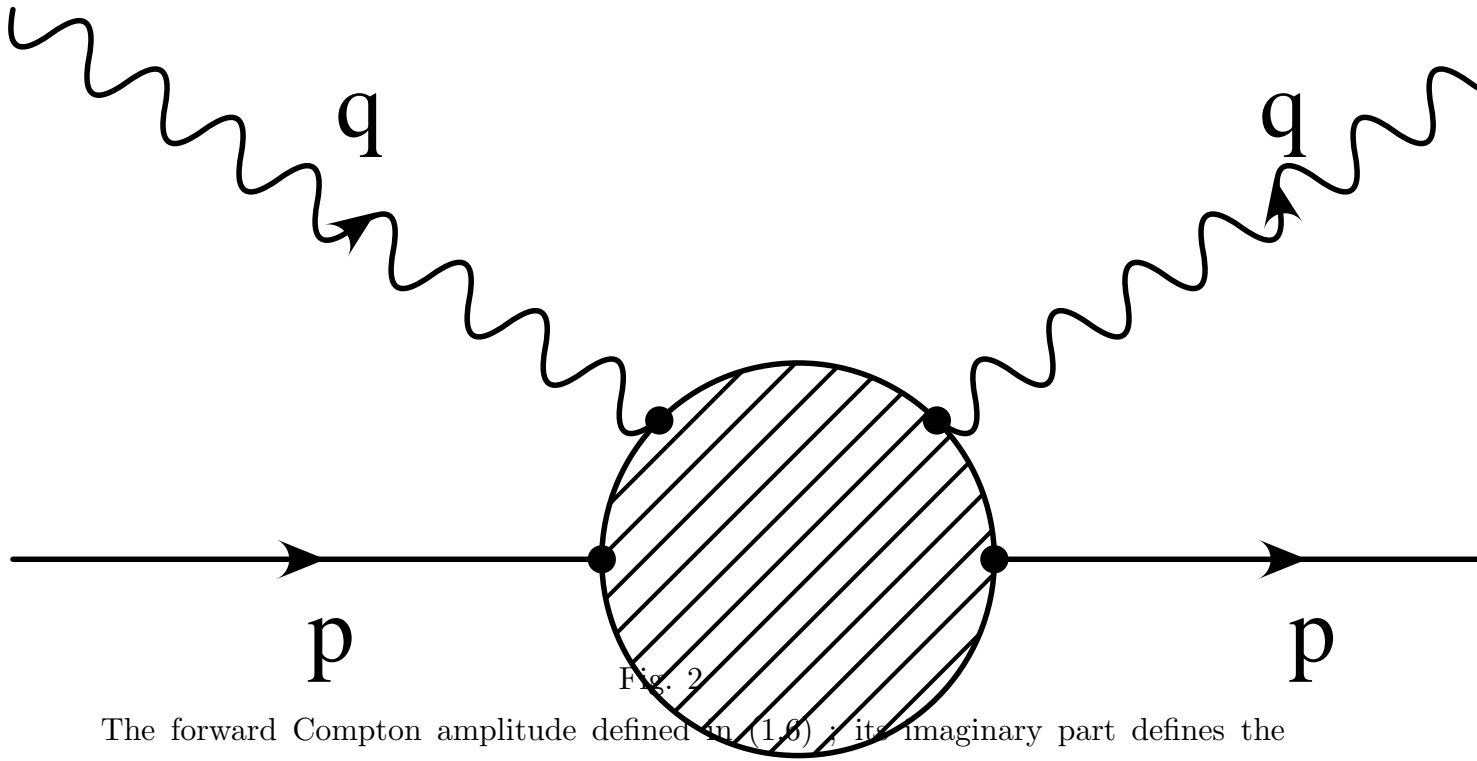
In this paper we have shown how scaling laws for deep inelastic electropion production derived on rather general grounds from QCD-inspired current algebra, are manifested in QCD perturbation theory. Leading logarithmic corrections are calculated and an evolution equation for the amplitude derived. These are quite similar in character to the well-known ones occurring in the conventional Compton amplitude but have the advantage that the predictions for the full amplitude are, in this case, amenable to experiment. In the Compton case, only the imaginary parts (the conventional deep inelastic structure functions) are, in practice, measurable. However, for the full amplitude we show that, contrary to one's naive expectation, the usual deviations from scaling derived from an operator product expansion analysis do not lead to a well-defined prediction in the physical region. Thus, unlike the structure function case, the QCD perturbation theory result cannot be “rigorously” justified from asymptotic freedom. The reason for this can be traced back to the behavior of the amplitude near $x \approx 0$; the OPE leads to an expansion in $1/x$ which cannot converge for a physical process. The conventional moment equations for the structure functions which exploit the known analytic properties of the amplitude are precisely designed to circumvent this difficulty. Thus, observation of the scaling laws and their violation for the full amplitude can potentially shed light on the small x behavior and help clarify just how far one can push results based on QCD perturbation theory.

With renewed interest in such problems stimulated by recent HERA results and the potential of detailed data from CEBAF (albeit at relatively low energies) we feel that it is important to examine processes such as these that are natural extensions of the canonical structure functions. It should be stressed that these processes should also be viewed as yielding complementary data on the quark-gluon structure of the nucleon. In future work we intend to explore this aspect of the problem in more detail; meanwhile, the main thrust of this paper is motivated by the desire to rekindle interest in such problems. In fact it was partly stimulated by a query along these lines from the experimentalist Bogdan Povh of the University of Heidelberg; one of us (GBW) would like to thank him for his original inquiry.

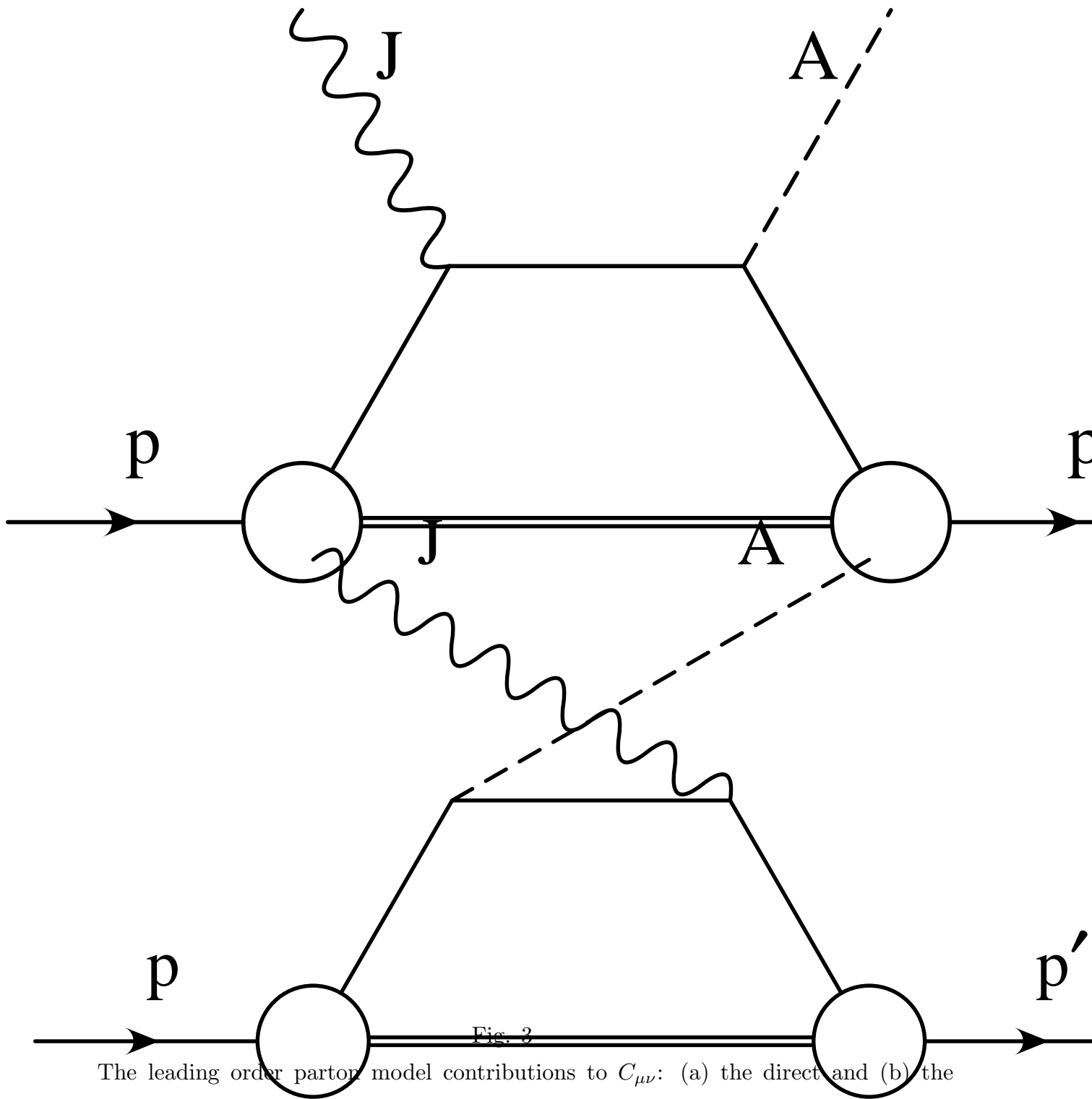
6. FIGURES



General electropion production amplitude defined in (1.1) showing the kinematics of the external particles.



The forward Compton amplitude defined in (1.6) ; its imaginary part defines the conventional structure functions, (1.5) .



The leading order parton model contributions to $C_{\mu\nu}$: (a) the direct and (b) the crossed contributions.

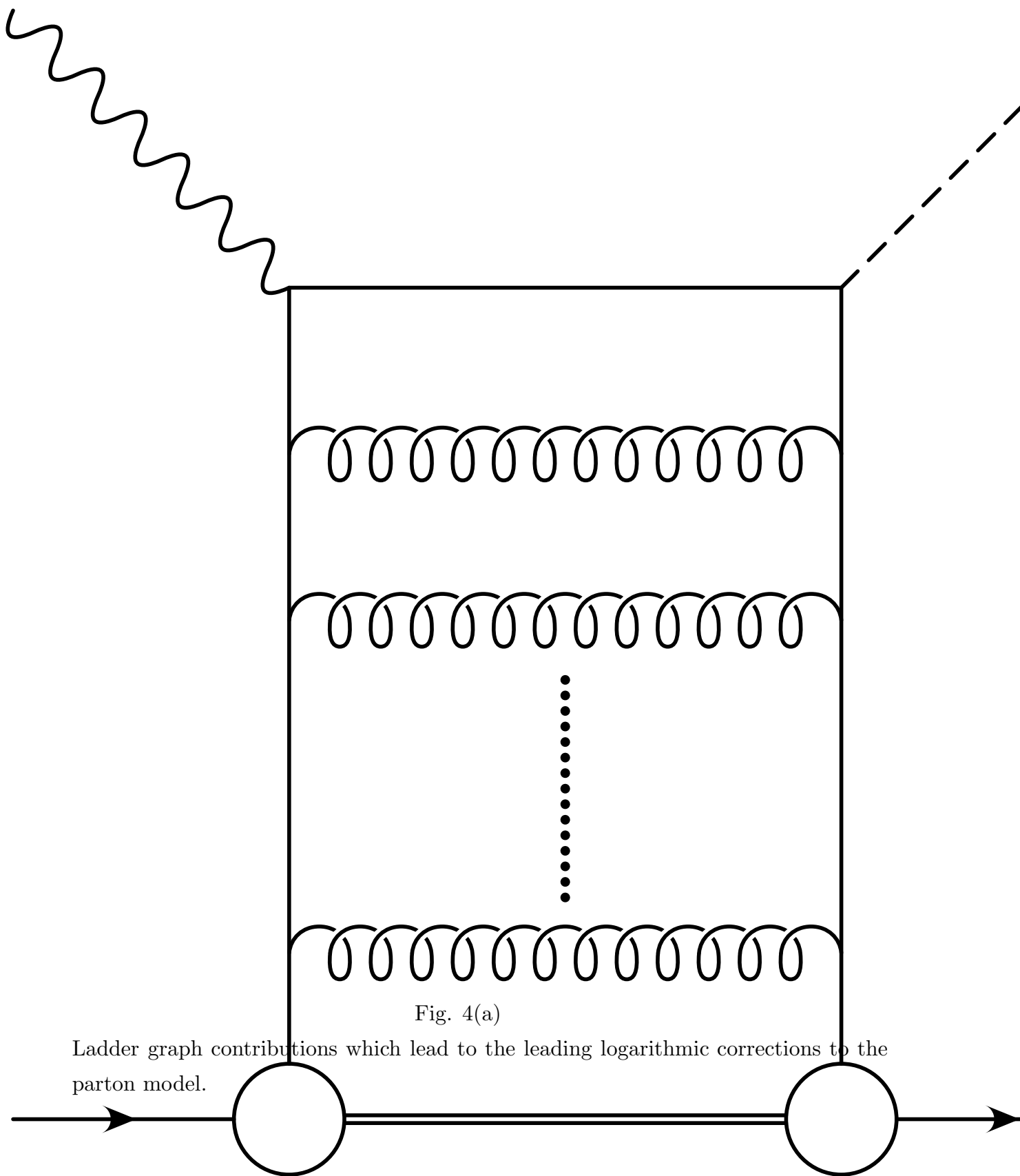


Fig. 4(a)

Ladder graph contributions which lead to the leading logarithmic corrections to the parton model.

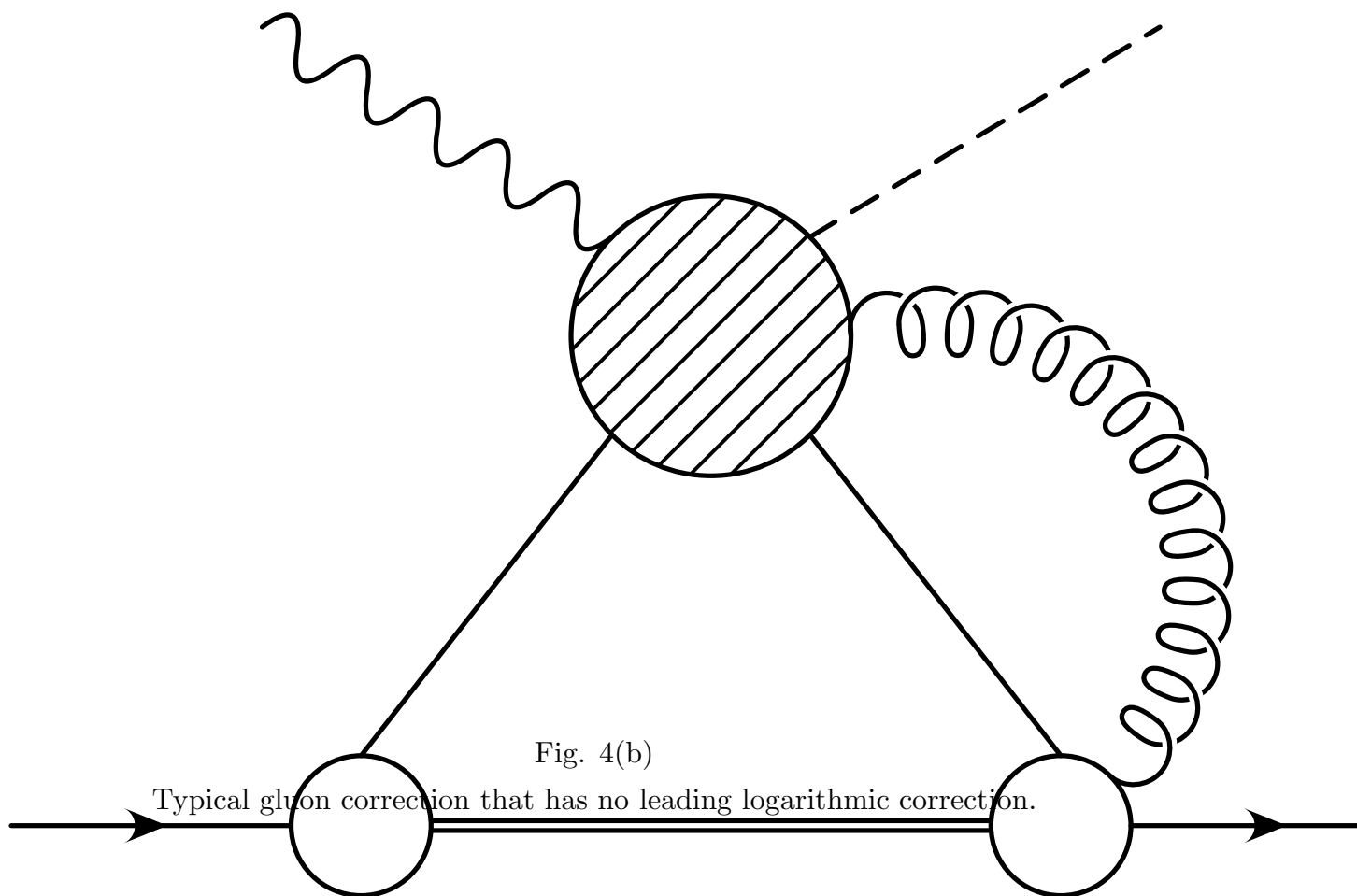


Fig. 4(b)

Typical gluon correction that has no leading logarithmic correction.

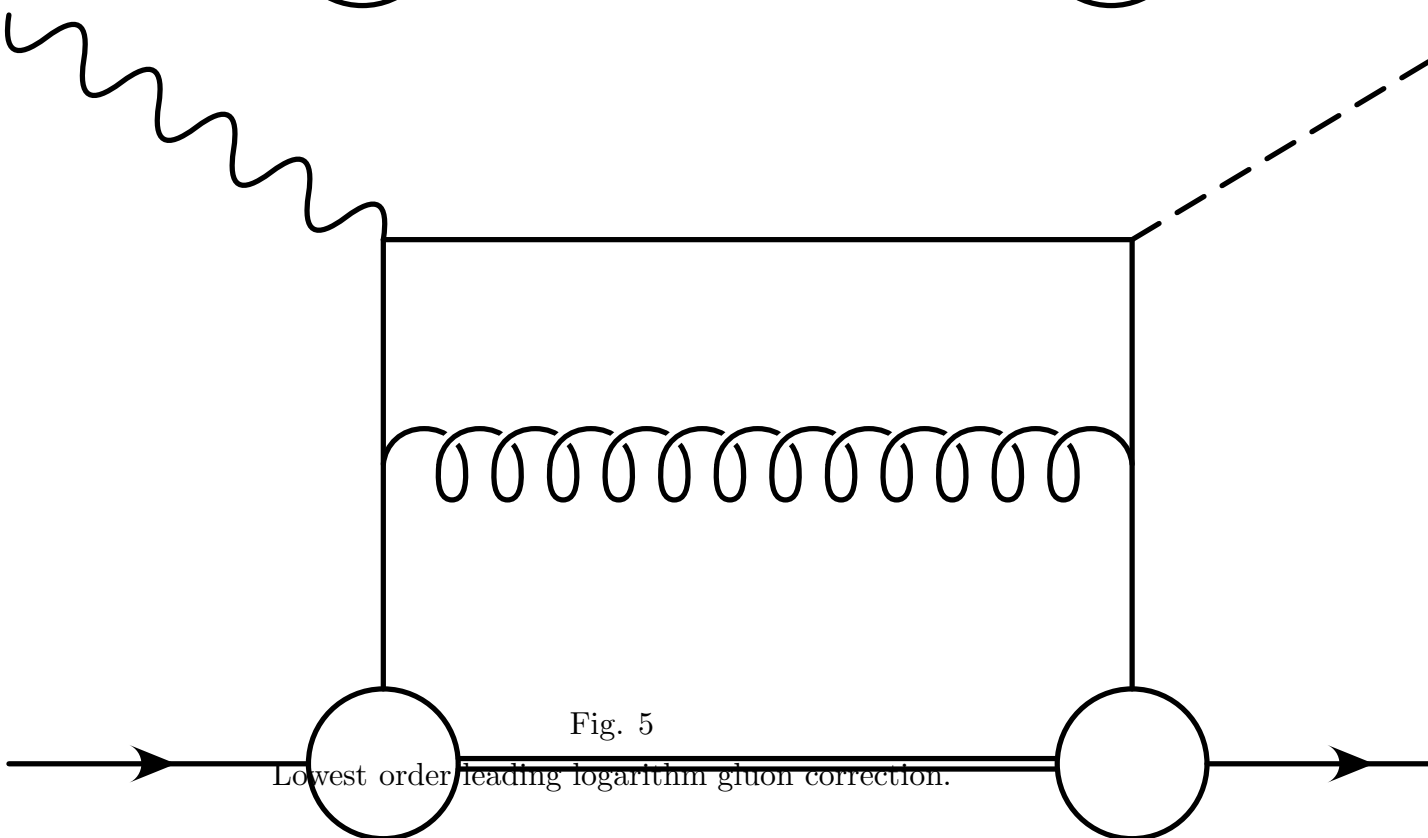


Fig. 5

Lowest order leading logarithm gluon correction.

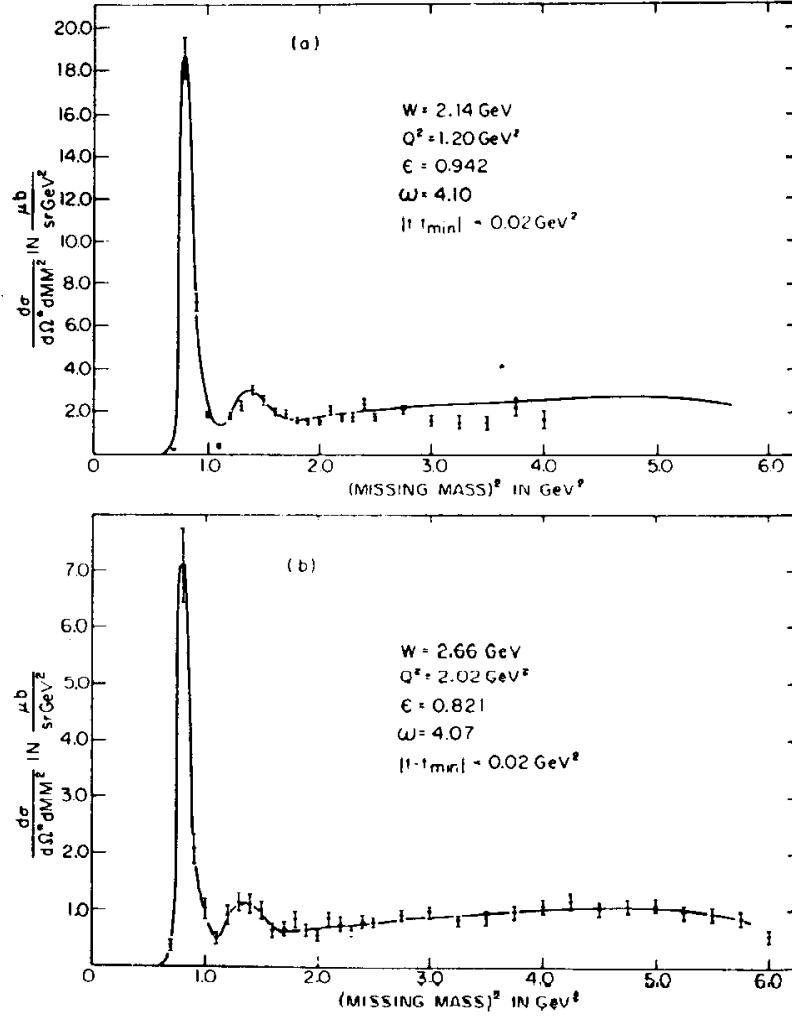


Fig. 6

The virtual photoproduction cross-section at two different values of $W = \sqrt{s}$ but at the same value of $\omega \equiv 1/x$; the data are taken from [9]. According to eqs. (4.1) and (4.2) these should be identical except for a scale factor W^4 .

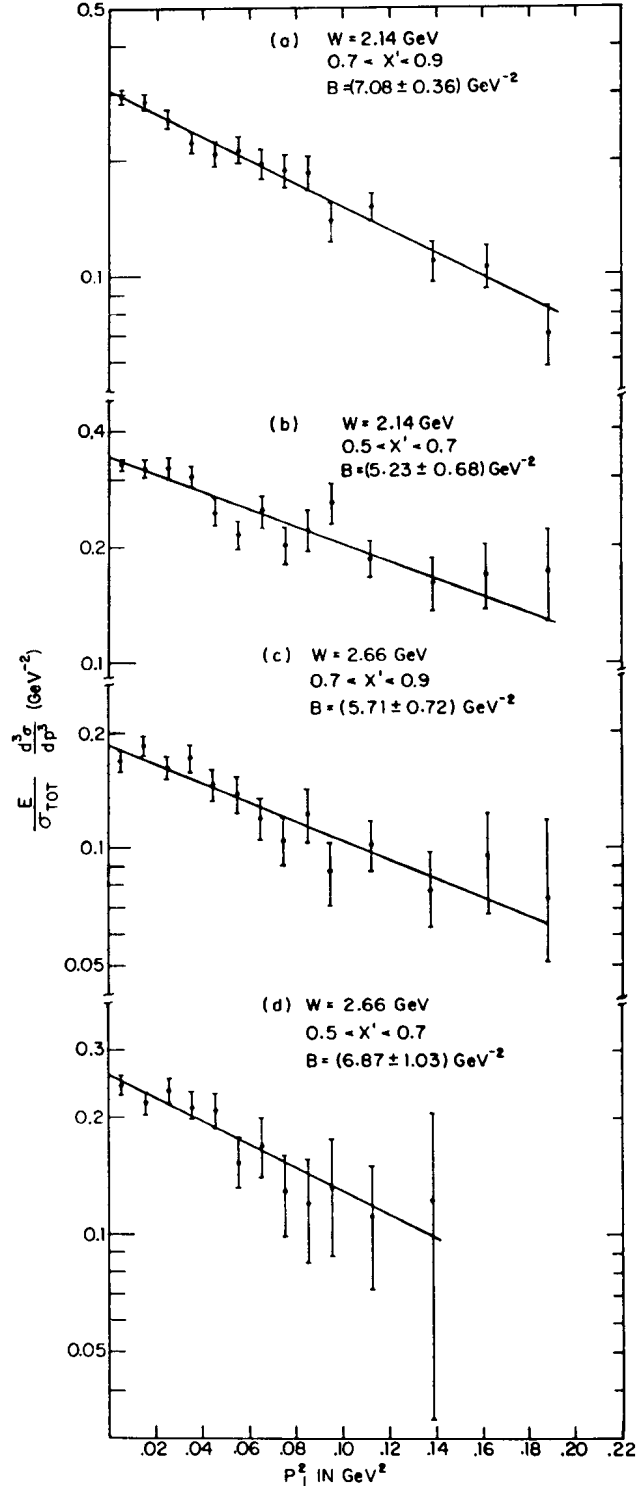


Fig. 7

Transverse-momentum distribution of the produced pions for two regions of longitudinal pion momentum. The similarity of the data at different values of W are in agreement with the scaling argument.

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